

Kaluza-Klein Reduction in a Warp Geometry Revisited

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Abstract

We study the Kaluza-Klein dimensional reduction of zero-modes of bulk antisymmetric tensor fields on a non-compact extra dimension in the Randall-Sundrum model. It is shown that in the Kaluza-Klein reduction on a non-compact extra dimension we have in general a zero-mode depending on a fifth dimension in addition to a conventional constant zero-mode in the Kaluza-Klein reduction on a circle. We examine the localization property of these zero-modes on a flat Minkowski 3-brane. In particular, it is shown that a 2-form and a 3-form on the brane can be respectively obtained from a 3-form and 4-form in the bulk by taking the zero-mode dependent on the fifth dimension.

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The gravity-localized models in a brane world have the intriguing feature that even if there is a non-compact extra dimension, the graviton is sharply localized on a flat Minkowski brane, thereby reproducing the four-dimensional Newton's law with negligible corrections on the brane [1, 2]. (For multi-brane models, see [3].) It is then natural to ask whether matter and gauge fields in addition to the graviton are also localized on the brane by a gravitational interaction. If the entire local fields are trapped on the brane, we could regard such a 3-brane as a candidate of our real world. Indeed, in superstring theory, matter and gauge fields are naturally confined to D3 branes due to open strings ending on the branes while the gravity is free to propagate in a bulk space-time due to closed strings living in the bulk [4]. On the other hand, in local field theory, it has been well known that except a 1-form gauge field all the local fields are also localized on a brane by a gravitational interaction [5, 6, 7, 8, 9, 10]. (For a review see [11].)

Recently, we have proposed a completely new mechanism for trapping a 1-form bulk gauge field on a flat brane in the gravity-localized model [12]. The key idea is to consider a topological term and a 3-form action in addition to the $U(1)$ gauge field action in order to give the mass term with a 'kink' profile to the bulk gauge field. (This mass generation mechanism is called 'topological mass generation' or 'topological Higgs mechanism' [13, 14, 15].) This new localization mechanism is very similar to that of fermions [16] in the sense that in both the cases the zero-modes share the same exponentially damping form and then some inequality between the bulk mass and the constant in the warp factor is needed to insure the localization and massless condition of brane gauge field at the same time.

Accordingly, it has been now shown that the entire bulk fields ranging from a spin-0 scalar to a spin-2 graviton are localized on a flat brane only by a gravitational interaction in the Randall-Sundrum model [1, 2]. The fields we have left aside in the above study are the antisymmetric tensor fields. Since many antisymmetric fields appear in the low energy effective action of superstring theory, we should also study the localization property of these fields on a flat 3-brane. Recently, it has been shown that a 2-form potential is also confined to a flat brane in the Randall-Sundrum model [17]. (See also closely related papers [18].) The important observation there is that a 3-form in a bulk yields a 2-form on a brane by the nontrivial Kaluza-Klein dimensional reduction, which has been found through the Hodge duality relation between a massless 0-form and a massless 3-form in the five-dimensional space-time. Recall that such a nontrivial zero mode depending on a fifth dimension has also been utilized in the study of the localization of bulk fields in the locally-localized gravity models [19].

The modest aim of the present paper is to clarify the Kaluza-Klein reduction of antisymmetric tensor fields on a non-compact extra dimension in the Randall-Sundrum model. We shall make use of a more general approach rather than an approach on the basis of the Hodge duality [17] since the latter approach relies heavily on the Hodge duality, which holds only on-shell. Because of it, for instance, provided that there are mass terms and topological terms such as Chern-Simons and BF terms, only the former approach provides us with a useful method. Actually, such a situation has been already appeared in Ref. [12]. For com-

pleteness, in this paper, we shall consider the entire antisymmetric tensor fields ranging from a 0-form scalar field to a 4-form potential in the five-dimensional Randall-Sundrum model. Incidentally the generalization to the non-abelian gauge fields and higher dimensions would be straightforward.

We shall start by fixing our model setup. The metric ansatz we take is of the Randall-Sundrum form [2]:

$$\begin{aligned} ds^2 &= g_{MN} dx^M dx^N \\ &= e^{-A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2, \end{aligned} \quad (1)$$

where M, N, \dots are five-dimensional space-time indices and μ, ν, \dots are four-dimensional brane indices. The metric on the brane $\eta_{\mu\nu}$ denotes the four-dimensional flat Minkowski metric with signature $(-, +, +, +)$. Moreover, $A(r) = 2k|r|$ where k is a positive constant and the fifth dimension r runs from $-\infty$ to ∞ . We have a model setup in mind where a single flat 3-brane sits at the origin $r = 0$ of the fifth dimension and various antisymmetric tensor fields reside in a bulk. We will assume that the background metric is not modified by the presence of the bulk fields, that is, we will neglect the back-reaction on the metric from the bulk fields. Under such a model setup, we look for zero-modes of the bulk fields in a simple Kaluza-Klein ansatz such that the zero-modes are not only normalizable but also localized sharply near the brane. Here it is worthwhile to stress one important point that the normalizable condition, which is equivalent to the convergence of the integral over the fifth dimension r in front of the kinetic terms, is usually thought to be a necessary and sufficient condition for the localization of the bulk fields on a brane [8]. However, as shown in Refs. [12, 19], we sometimes meet the situation where the zero-modes are normalizable but spread rather widely in a bulk. Perhaps such the widely spread zero-modes would be in contradiction with experimental results such as the charge conservation law. Thus, in order to show a complete localization of bulk fields on a brane, we have to check that the normalized zero-modes in a flat space take an exponentially damping form in addition to the normalizable condition.

We shall start with a massless 0-form real scalar field [8]. The action of a 0-form potential Φ is given by

$$S_0 = -\frac{1}{2} \int d^5x \sqrt{-g} g^{MN} \partial_M \Phi \partial_N \Phi. \quad (2)$$

Then the equation of motion becomes

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) = 0. \quad (3)$$

As mentioned before, we shall look for a zero-mode solution with the following simple Kaluza-Klein ansatz

$$\Phi(x^M) = \phi(x^\mu) u(r), \quad (4)$$

where we assume the equation of motion for the brane scalar field, $\square \phi = 0$ with being $\square \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu$.

With the ansatz (4), Eq. (3) reduces to a single differential equation for $u(r)$:

$$\partial_r \left(e^{-2A(r)} \partial_r u \right) = 0. \quad (5)$$

The general solution to this equation is easily found to be

$$u(r) = \frac{u_1}{4k} e^{2A(r)} + u_0, \quad (6)$$

where u_0 and u_1 are integration constants. To derive this solution, we have assumed that $\partial_r u(0) = 0$, which would stem from the fact that if we impose Z_2 orbifold symmetry on the fifth dimension, the self-adjointness of the differential operator requires the Neumann boundary condition at $r = 0$. Henceforth we will also assume $\partial_r u(0) = 0$.

Here we wish to emphasize an important fact associated with the Kaluza-Klein reduction on a non-compact dimension in a warp geometry, which will indeed play a critical role in deriving zero-modes bound to a brane from fields in a bulk. In the ordinary Kaluza-Klein dimensional reduction scenario on a compact circle, a zero-mode is just a constant since there is a cyclic symmetry $r \rightarrow r + 2\pi$ and usually a zero-mode respecting this symmetry is only a constant. (Of course, the excited modes have a factor e^{inr} with n being integers, which obviously respects the cyclic symmetry.) On the other hand, in a non-compact case at hand, we have no room to impose such a symmetry once the position of a brane is fixed, so that a zero-mode with the nontrivial dependence on an extra dimension is allowed.

Plugging the ansatz (4) into the starting action (2), the action can be cast to the form

$$S_0^{(0)} = \int d^4x dr \left[-\frac{1}{2} \partial^\mu \phi \partial_\mu \phi e^{-A(r)} u^2(r) - \frac{1}{2} \phi^2 e^{-2A(r)} (\partial_r u)^2 \right]. \quad (7)$$

Here we take a special solution $u(r) = u_0$, which is a constant zero-mode, since the solution has a more convergent character than the solution with the nonzero u_1 . Then, the integral over r in front of the kinetic term in Eq. (7) is found to be finite as follows:

$$I \equiv \int_{-\infty}^{\infty} dr e^{-A(r)} u^2(r) = \frac{u_0^2}{k}. \quad (8)$$

This convergence of the r -integral implies the normalizability of the zero mode $u(r) = u_0$ of a bulk scalar field. In order to show a sharp localization, it is necessary to check that the normalized zero-mode has an exponentially damping form [12]. Actually, the normalized zero-mode in a flat space has the form

$$\hat{u}(r) = \frac{1}{\sqrt{I}} e^{-\frac{1}{2}A(r)} u(r) = \sqrt{k} e^{-k|r|}, \quad (9)$$

which obviously indicates that the brane scalar field is sharply localized near the brane sitting at $r = 0$ as long as $k \gg 1$.

Next, we shall turn to a massless 1-form potential, that is, a $U(1)$ gauge field. The path of arguments is very similar to the case of a 0-form potential. The action is

$$S_1 = -\frac{1}{4} \int d^5x \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS}, \quad (10)$$

where $F_{MN} = 2\partial_{[M}A_{N]} = \partial_MA_N - \partial_NA_M$. The equations of motion read

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}g^{RS}F_{NS}\right) = 0. \quad (11)$$

We search for a zero-mode solution of the form

$$\begin{aligned} A_\mu(x^M) &= a_\mu(x^\lambda)u(r), \\ A_r(x^M) &= a(x^\lambda)v(r), \end{aligned} \quad (12)$$

where we assume the equations of motion on a brane, $\partial^\nu f_{\mu\nu} = \square a = 0$ with the definition of $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. (Here we have not fixed the gauge symmetries in five dimensions.)

With this ansatz, Eq. (11) reduces to two differential equations for $u(r)$ and $v(r)$:

$$\begin{aligned} \partial_r\left(e^{-A(r)}\partial_ru\right) &= 0, \\ \partial_r\left(e^{-A(r)}v\right) &= 0, \end{aligned} \quad (13)$$

whose general solution is given by

$$\begin{aligned} u(r) &= \frac{u_1}{2k}e^{A(r)} + u_0, \\ v(r) &= v_0e^{A(r)}. \end{aligned} \quad (14)$$

where u_0 , u_1 and v_0 are integration constants. From the better convergent property, we shall choose the solution with $u_1 = 0$ in what follows.

Plugging this solution into the classical action (10), the action takes the form

$$S_1^{(0)} = \int d^4x dr \left[-\frac{1}{4}f_{\mu\nu}^2 u_0^2 - \frac{1}{2}\partial_\mu a \partial^\mu a e^{A} v_0^2 \right]. \quad (15)$$

Note that the two r -integrals in front of the vector and scalar kinetic terms, those are, $I_1 \equiv \int_{-\infty}^{\infty} dr u_0^2$ and $I_2 \equiv \int_{-\infty}^{\infty} dr e^A v_0^2$ diverge, thereby implying that a 1-form a_μ and a 0-form a are not localized on a brane, which is the well-known fact. This problem has been recently circumvented by coupling the bulk gauge field to a 3-form potential through a topological term [12].

We are now ready to consider a 2-form potential, in other words, the Kalb-Ramond second-rank antisymmetric tensor field. In five dimensions, a 2-form is dual to a 1-form, so it is expected that a 2-form is also trapped on a brane as in a 1-form. However, it is impossible to derive the 2-form confined to the brane from the conventional action of a 2-form. We are now familiar with the fact that such a trapped 2-form can be obtained from the action of a 3-form through the Kaluza-Klein reduction on a non-compact extra dimension [17].

The classical action of a massless 2-form is given by

$$S_2 = -\frac{1}{12} \int d^5x \sqrt{-g} g^{M_1N_1} g^{M_2N_2} g^{M_3N_3} F_{M_1M_2M_3} F_{N_1N_2N_3}, \quad (16)$$

where $F_{MNP} = 3\partial_{[M}A_{NP]} = \partial_MA_{NP} + \partial_NA_{PM} + \partial_PA_{MN}$. The equations of motion are

$$\frac{1}{\sqrt{-g}}\partial_{M_3}\left(\sqrt{-g}g^{M_1N_1}g^{M_2N_2}g^{M_3N_3}F_{N_1N_2N_3}\right) = 0. \quad (17)$$

The natural Kaluza-Klein ansatz for a zero-mode solution is

$$\begin{aligned} A_{\mu\nu}(x^M) &= a_{\mu\nu}(x^\lambda)u(r), \\ A_{r\mu}(x^M) &= a_\mu(x^\lambda)v(r), \end{aligned} \quad (18)$$

where the following equations on a brane are imposed: $\partial^\rho f_{\mu\nu\rho} = \partial^\nu f_{\mu\nu} = 0$ with the definition being $f_{\mu\nu\rho} = 3\partial_{[\mu}a_{\nu\rho]}$.

With this ansatz, Eq. (17) reduces to differential equations:

$$\begin{aligned} \partial_r^2 u &= 0, \\ \partial_r v &= 0, \end{aligned} \quad (19)$$

whose general solution is simply given by

$$\begin{aligned} u(r) &= u_1 r + u_0, \\ v(r) &= v_0. \end{aligned} \quad (20)$$

Let us select a solution with $u_1 = 0$ because of the better convergent property.

Substituting this solution into the action (16), the action reduces to the form

$$S_2^{(0)} = \int d^4x dr \left[-\frac{1}{12}f_{\mu\nu\rho}^2 e^A u_0^2 - \frac{1}{4}f_{\mu\nu}^2 v_0^2 \right]. \quad (21)$$

Note that the two integrals over r , those are, $I_1 \equiv \int_{-\infty}^{\infty} dr e^A u_0^2$ and $I_2 \equiv \int_{-\infty}^{\infty} v_0^2$ diverge, so that neither a 2-form $a_{\mu\nu}$ nor a 1-form a_μ are localized on a brane. Therefore, starting with the action of a 2-form in a bulk we cannot obtain a 2-form bound to a brane.

Here let us comment on the possibility of generalizing a new mechanism for trapping of a 1-form to the case of a 2-form [12]. Perhaps, the most plausible possibility would be to consider the following action:

$$S_2 = \int \left[-\frac{1}{2}B_2 \wedge *B_2 - \frac{1}{2}C_2 \wedge *C_2 + mB_2 \wedge dC_2 \right], \quad (22)$$

where B_2 and C_2 are two 2-form fields, and we have used the form notations for convenience. With the gauge conditions $B_{rM} = C_{rM} = 0$, we have analyzed the localization property of $B_{\mu\nu}$ and $C_{\mu\nu}$. Unfortunately, it turns out that these two 2-forms have normalizable zero-modes but are not localized sharply on a flat 3-brane, as in the locally-localized gravity models [19].

Let us turn our attention to a 3-form potential. We will see that a 3-form in a bulk yields a 2-form trapped on a brane via the Kaluza-Klein reduction. The classical action of a massless 3-form potential reads

$$S_3 = -\frac{1}{48} \int d^5x \sqrt{-g} g^{M_1N_1} g^{M_2N_2} g^{M_3N_3} g^{M_4N_4} F_{M_1M_2M_3M_4} F_{N_1N_2N_3N_4}, \quad (23)$$

where $F_{MNPQ} = 4\partial_{[M}A_{NPQ]} = \partial_M A_{NPQ} - \partial_N A_{MPQ} + \partial_P A_{MNQ} - \partial_Q A_{MNP}$. The equations of motion are then

$$\frac{1}{\sqrt{-g}}\partial_{M_4}\left(\sqrt{-g}g^{M_1N_1}g^{M_2N_2}g^{M_3N_3}g^{M_4N_4}F_{N_1N_2N_3N_4}\right) = 0. \quad (24)$$

We shall make the Kaluza-Klein ansatz

$$\begin{aligned} A_{\mu\nu\rho}(x^M) &= a_{\mu\nu\rho}(x^\lambda)u(r), \\ A_{r\mu\nu}(x^M) &= a_{\mu\nu}(x^\lambda)v(r), \end{aligned} \quad (25)$$

where we assume that $\partial^\sigma f_{\mu\nu\rho\sigma} = \partial^\rho f_{\mu\nu\rho} = 0$ with the definition being $f_{\mu\nu\rho\sigma} = 4\partial_{[\mu}a_{\nu\rho\sigma]}$.

Then Eq. (24) becomes

$$\begin{aligned} \partial_r(e^A\partial_r u) &= 0, \\ \partial_r(e^A v) &= 0. \end{aligned} \quad (26)$$

The general solution is given by

$$\begin{aligned} u(r) &= -\frac{u_1}{2k}e^{-A} + u_0, \\ v(r) &= v_0e^{-A}. \end{aligned} \quad (27)$$

Inserting the ansatz (25) to the starting action (23), we have

$$S_3^{(0)} = \int d^4x dr \left[-\frac{1}{48}f_{\mu\nu\rho\sigma}^2 e^{2A}u^2 - \frac{1}{12}(a_{\mu\nu\rho}\partial_r u - f_{\mu\nu\rho}v)^2 e^A \right]. \quad (28)$$

We shall discuss the two cases separately, one of which is $u_1 = 0$ and the other is $u_1 \neq 0$.

In the case of $u(r) = u_0$ (i. e., $u_1 = 0$), the above action takes the form

$$S_3^{(0)} = \int d^4x dr \left[-\frac{1}{48}f_{\mu\nu\rho\sigma}^2 e^{2A}u_0^2 - \frac{1}{12}f_{\mu\nu\rho}^2 e^A v^2 \right]. \quad (29)$$

The integral over r in front of the kinetic term for a 3-form $a_{\mu\nu\rho}$, $I_1 \equiv \int_{-\infty}^{\infty} dr e^{2A}u_0^2$ diverges, so the 3-form is not localized on a brane. On the other hand, the integral in front of the kinetic term for a 2-form $a_{\mu\nu}$, $I_2 \equiv \int_{-\infty}^{\infty} e^A v^2$ takes the finite value of $\frac{v_0^2}{k}$. This fact suggests that the 2-form might be localized near the brane. Indeed, the normalized zero-mode for the 2-form in a flat space is given by

$$\hat{v}(r) = \frac{1}{\sqrt{I_2}}e^{\frac{1}{2}A(r)}v(r) = \sqrt{k}e^{-k|r|}, \quad (30)$$

which shows that the 2-form $a_{\mu\nu}$, which has been obtained from a bulk 3-form via the Kaluza-Klein reduction, is sharply localized on a brane as long as $k \gg 1$. Incidentally, the zero-mode

for the 3-form has a behavior like $\hat{u}(r) \sim e^A u_0 = e^{2k|r|} u_0$, so the 3-form resides in a bulk away from a brane. Hence, effectively on a brane, we have an action

$$S_3^{(0)} = -\frac{1}{12} \int d^4x f_{\mu\nu\rho}^2, \quad (31)$$

where we have redefined as $\frac{v_0}{\sqrt{k}} a_{\mu\nu} \rightarrow a_{\mu\nu}$.

Next, let us consider the case $u(r) = -\frac{u_1}{2k} e^{-A} + u_0$ (i. e., $u_1 \neq 0$). We shall note first that in the case of $u(r) = -\frac{u_1}{2k} e^{-A} + u_0$, we can rewrite a term in (28) as follows:

$$a_{\mu\nu\rho} \partial_r u - f_{\mu\nu\rho} v = (a_{\mu\nu\rho} - \frac{v_0}{u_1} f_{\mu\nu\rho}) u_1 e^{-A}. \quad (32)$$

With the field redefinition (or equivalently, the gauge transformation)

$$a_{\mu\nu\rho} \rightarrow a_{\mu\nu\rho} + \frac{v_0}{u_1} f_{\mu\nu\rho}, \quad (33)$$

the action transforms as

$$S_3^{(0)} = \int d^4x dr \left[-\frac{1}{48} f_{\mu\nu\rho\sigma}^2 e^{2A} u^2 - \frac{1}{12} a_{\mu\nu\rho}^2 e^A (\partial_r u)^2 \right]. \quad (34)$$

In this action, the integral over r in front of the kinetic term obviously diverges, so that a 3-form $a_{\mu\nu\rho}$ is not localized on a brane, but resides in a bulk. The real problem here is that this time we have no a 2-form $a_{\mu\nu}$ on the brane, which should be in contrast to the previous $u(r) = u_0$ case, where a 2-form stemming from a 3-form is bound to the brane. To avoid this difficulty, Duff and Liu have chosen $a_{\mu\nu\rho} = 0$ from the beginning although their reasoning is completely different from ours and is based on the Hodge duality [17]. It is then natural to ask why the results between ours and Duff et al. appear to be so different. The answer lies in the gauge symmetries in the starting action (23). The action (23) has the gauge symmetries as well as off-shell reducible symmetries [20]:

$$\begin{aligned} \delta A_{MNP} &= \partial_{[M} \varepsilon_{NP]}, \\ \delta \varepsilon_{MN} &= \partial_{[M} \varepsilon_{N]}, \\ \delta \varepsilon_M &= \partial_M \varepsilon, \end{aligned} \quad (35)$$

whose number of degrees of freedom is $\frac{5 \times 2}{2} - 5 + 1 = 6$, which exactly coincides with the number of dynamical degrees of freedom involved in $A_{r\mu\nu}$. Accordingly, $a_{\mu\nu}$ can be gauge-fixed to be zero, which yields our result (34), in other words, no 2-form on a brane. From this viewpoint, the result by Duff et al. is interpreted as follows: They have picked up the gauge conditions $A_{\mu\nu\rho} = 0$ from the outset, so $a_{\mu\nu}$ cannot be gauged away any more and consequently appears in the final action (31).

Finally, we would like to consider a massless 4-form field. This field is non-dynamical but it is of theoretical interest to check if the approach at hand also applies to this case. It is expected that such a non-dynamical field might play an important role in the cosmological

constant problem, so there would be also of some physical importance. The classical action of a massless 4-form reads

$$S_4 = -\frac{1}{240} \int d^5x \sqrt{-g} g^{M_1 N_1} g^{M_2 N_2} g^{M_3 N_3} g^{M_4 N_4} g^{M_5 N_5} F_{M_1 M_2 M_3 M_4 M_5} F_{N_1 N_2 N_3 N_4 N_5}, \quad (36)$$

where $F_{MNPQR} = 5\partial_{[M} A_{NPQR]}$. The equations of motion then take the form

$$\partial_{M_5} \left(\sqrt{-g} g^{M_1 N_1} g^{M_2 N_2} g^{M_3 N_3} g^{M_4 N_4} g^{M_5 N_5} F_{N_1 N_2 N_3 N_4 N_5} \right) = 0. \quad (37)$$

Making the Kaluza-Klein ansatz

$$\begin{aligned} A_{\mu\nu\rho\sigma}(x^M) &= a_{\mu\nu\rho\sigma}(x^\lambda)u(r), \\ A_{r\mu\nu\rho}(x^M) &= a_{\mu\nu\rho}(x^\lambda)v(r), \end{aligned} \quad (38)$$

with the assumption that $\partial^\sigma f_{\mu\nu\rho\sigma} = \partial^\sigma a_{\mu\nu\rho\sigma} = 0$, the action reads

$$S_4^{(0)} = -\frac{1}{48} \int d^4x dr (a_{\mu\nu\rho\sigma} \partial_r u - f_{\mu\nu\rho\sigma} v)^2 e^{2A}. \quad (39)$$

Moreover, Eq. (37) becomes

$$\begin{aligned} \partial_r (e^{2A} \partial_r u) &= 0, \\ \partial_r (e^{2A} v) &= 0. \end{aligned} \quad (40)$$

The general solution is given by

$$\begin{aligned} u(r) &= -\frac{u_1}{4k} e^{-2A} + u_0, \\ v(r) &= v_0 e^{-2A}. \end{aligned} \quad (41)$$

Again, we shall first consider the case of $u(r) = u_0$, that is, $u_1 = 0$. Then, the action (39) reduces to

$$S_4^{(0)} = -\frac{1}{48} \int d^4x dr f_{\mu\nu\rho\sigma}^2 e^{2A} v^2. \quad (42)$$

Since $I \equiv \int_{-\infty}^{\infty} dr e^{2A} v^2 = \frac{v_0^2}{2k}$, this zero-mode is a normalizable one. In fact, the normalized zero-mode is given by $\hat{v}(r) = \sqrt{2k} e^{-2k|r|}$, so a 3-form $a_{\mu\nu\rho}$ is localized on a brane.

Next, let us consider the case of $u(r) = -\frac{u_1}{4k} e^{-2A} + u_0$. This time, the field redefinition

$$a_{\mu\nu\rho\sigma} \rightarrow a_{\mu\nu\rho\sigma} + \frac{v_0}{u_1} f_{\mu\nu\rho\sigma}, \quad (43)$$

leads to

$$\begin{aligned} S_4^{(0)} &= -\frac{1}{48} \int d^4x dr a_{\mu\nu\rho\sigma}^2 e^{2A} (\partial_r u)^2 \\ &= -\frac{1}{48} \int d^4x \frac{u_1^2}{2k} a_{\mu\nu\rho\sigma}^2. \end{aligned} \quad (44)$$

Hence, as in a 3-form potential, in order to reproduce a 3-form on a brane, we need to fix the gauge symmetries by $A_{\mu\nu\rho\sigma} = 0$.

In conclusion, we have examined the Kaluza-Klein reduction on a non-compact dimension of bulk antisymmetric tensor fields from a 0-form to a 4-form in the Randall-Sundrum model. Compared with the ordinary Kaluza-Klein reduction on a compact circle, we can find zero-modes which are manifestly dependent on the extra dimension. We have seen that such nontrivial zero-modes provide us with a 2-form and a 3-form bound to a brane from a 3-form and a 4-form in a bulk, respectively, through the Kaluza-Klein reduction. This observation has also been used in showing the localization of various bulk fields in the locally-localized gravity models [19].

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